Flowing maps to minimal surfaces

Melanie Rupflin and Peter Topping

We explore the idea of defining a geometric evolution equation that deforms a given surface into a minimal surface, that is into a critical point of the area functional.

The flow we define has elements in common with two well known parabolic evolution equations; the mean curvature flow which is the negative gradient flow of the area and the harmonic map heat flow which is the negative gradient flow of the Dirichlet energy. Using a combination of these two ideas has the advantage that we can profit of the much better analytical properties of the energy and thus of the harmonic map heat flow while still getting an object in the limit $t \to \infty$ that is not only a critical point of the energy (i.e. harmonic) but indeed a critical point of the area (i.e. minimal).

We recover the classical results of Sacks-Uhlenbeck and Schoen-Yau about the existence of (branched) minimal immersions, originally proved by direct minimisation technique, with the (branched) minimal immersion now obtained as the limit of the flow starting from any admissible initial data.