On the compactness of solutions to multidimensional conservation law with discontinuous flux

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We consider the following Cauchy problem for multidimensional scalar conservation law

$$u_t + \mathrm{div} f(x,u) = 0, \qquad u(x,0) = u_0(x),$$

where $u = u(t,x)$, $x \in \mathbf{R}^d$, $t \in \mathbf{R}^+$ and $f = (f_1, ..., f_d) : \mathbf{R}^{d+1} \to \mathbf{R}^d$, $d \in \mathbf{N}$. For the initial data u_0 we assume that $u_0 \in (BV \cap L^\infty)(\mathbf{R}^d)$, $\quad a \le u_0(x,y) \le b, \quad x \in \mathbf{R}^d$. The flux function f has the following properties: $f_i(\cdot, \lambda) \in (BV \cap L^\infty)(\mathbf{R}^d)$, for every $\lambda \in \mathbf{R}$, $f_i(x, \cdot) \in Lip(\mathbf{R})$, for every $x \in \mathbf{R}^d$, $0 = f(x,b) = f(x,a)$, for every $x \in \mathbf{R}^d$.

We analyze a family of solutions to a regularization of the mentioned problem by smoothing flux function and initial data and involving the vanishing viscosity. We present a new genuine nonlinearity condition, weaker then in previous works on the subject, and prove strong L^1_{loc}-precompactness of mentioned family of solutions.

References

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