Dynamic and Stochastic Metastability: Problems at the boundary of applied analysis, PDE, and probability theory

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In this talk we describe two distinct and intriguing phenomena in applied analysis: dynamic metastability and stochastic metastability. Dynamic metastability of a dynamical system refers to the phenomenon in which the system relaxes quickly to a so-called dynamically metastable state, a lower-dimensional subspace of phase space that is not stable, but such that the evolution within this subspace is very slow. After a long time in or near the metastable state, the system may undergo drastic change, moving to a part of phase space that is far away. Hence, from both a physical and mathematical point of view, it is important to distinguish metastable states. Moreover, we need new analytical and computational tools to understand them. We describe recent progress.

Stochastic metastability, on the other hand, refers to noise-induced exit from a stable state. Here the underlying deterministic dynamics is relaxation to a stable state, but the noise occasionally induces a so-called rare event in which the system goes against the deterministic flow. What is the probability of such an event? Which kinds of random paths are most likely to be observed? The theory of large deviations and its extensions provide a link between these questions from probability and tools from analysis.