Young Women in Probability 2014
Bonn, 26th – 28th May 2014

Kumaraswamy Generalized Exponential Weibull Distribution

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Abstract

The Weibull distribution is perhaps the most widely applied statistical distribution for the modelling of the natural hazards. In this study Kumaraswamy-generalized exponential-Weibull (KGEW) distribution is introduced that generalizes the exponential Weibull (EW) distribution. The beauty and importance of this distribution lies in its ability to model monotone and non-monotone failure rate functions, which are quite common in the environmental studies.

The new distribution has a number of well-known lifetime special sub-models such as exponential, Weibull, Rayleigh distributions. The density function and study some properties of the new distribution are obtained by deriving survival and hazard function. The parameter estimation is implemented by using the maximum likelihood approaches. Then, we obtain the Fisher information matrix for the new model.

As an application, landslides that occurred in Turkey between 1900 and 2013 are studied. The results of an application to the landslide data indicate good performance of the proposed model.

Introduction

A natural hazard is an event of unusual magnitude that people do not expect and cannot control. They threaten people's lives and their activities and can forever change their ways of living. Natural hazards can become a natural disaster due to the destruction of people's property or their injury and/or death (Shield 2008). During the last decade, disasters caused an estimated average of US$67 billion/year in damage, with a maximum of US$230 billion, and a minimum of US$28 billion worldwide. Landslides are one of the most destructive natural hazards in the world, and casualties and property damage caused by landslides are high (Schuster, 1996). For this reason, there is an increasing international interest in landslide assessments and techniques (Aleotti and Chowdhury, 1999). The classical distributions frequently encountered for the inter-event time of the natural hazards are the exponential, Weibull, gamma distributions (Emanuel 2005).

The Weibull distribution is a very popular distribution for modeling natural hazard data. When modeling monotone hazard rates, the Weibull distribution may be an initial choice because of its negatively and positively skewed density shapes. However, it does not provide a reasonable parametric fit for modeling phenomenon with non-monotone failure rates such as the bathtub shaped and the unimodal failure rates that are common in reliability and biological studies. Such bathtub hazard curves have nearly flat middle portions and the corresponding densities have a positive anti-mode. In the last few years, new classes of distributions were proposed by extending the Weibull distribution to cope with bathtub shaped failure rates. A good review of some of these models is presented in Pham and Lai (2007). Between these, the exponentiated Weibull distribution introduced by Mudholkar et al. (1996), the additive Weibull distribution (Xie and Lai, 1995), the modified Weibull distribution proposed by Lai et al. (2003), the generalized modified Weibull distribution by Carrasco et al. (2008) and the beta modified Weibull distribution by Silva et al. (2009). Recently, EW distribution is proposed by Cordeiro et al. (2013).

In this study, we generalize the EW distribution using the Kumaraswamy-generalized distribution in order to gain a good picture of the landslides that occurred in Turkey. We refer to this distribution as KGEW.

KGEW Distribution

The random variable X has the EW distribution if its cumulative distribution function (cdf) and probability distribution (pdf) are given by respectively

$$G(x) = 1 - e^{-(\alpha x + \beta x^{\gamma})}, \qquad (1)$$
$$g(x) = (\alpha + \beta\gamma x^{\gamma-1}) e^{-(\alpha x + \beta x^{\gamma})}, \quad x > 0. \qquad (2)$$

where $\alpha > 0$ and $\gamma > 0$ are shape parameters and $\beta > 0$ is a scale parameter. Clearly, the exponential and Weibull distributions are special cases for $\beta = 0$ and $\alpha = 0$ respectively. The Rayleigh distribution arises when $\alpha = 0$ and $\gamma = 2$.

In this study, we propose an extension of the EW distribution based on the family of Kumaraswamy generalized distributions introduced by Cordeiro and de Castro (2011). cdf and pdf are given by respectively

$$F(x) = 1 - (1 - G(x)^{a})^{b}, \qquad (3)$$
$$f(x) = abG(x)^{a-1}(1 - G(x)^{a})^{b-1}. \qquad (4)$$

Here $G(x)$ is the cdf of X. $a > 0$ and $b > 0$ are parameters whose role is to introduce asymmetry and generate a distribution with heavier tails. If G(x) is the EW cumulative distribution, then (1) yields KGEW cumulative distribution for x > 0

$$F(x) = 1 - [1 - (1 - e^{-(\alpha x + \beta x^{\gamma})})^{a}]^{b},$$

$$f(x) = ab(\alpha + \beta\gamma x^{\gamma-1}) e^{-(\alpha x + \beta x^{\gamma})} [1 - e^{-(\alpha x + \beta x^{\gamma})}]^{a-1} [1 - (1 - e^{-(\alpha x + \beta x^{\gamma})})^{a}]^{b-1}$$

The KGEW distribution given above is much more flexible than the EW distribution and can allow for greater flexibility of tails. If X has a KGEW distribution, then the survival function S(x) c and hazard function h(x) can be expressed as

$$S(x) = [1 - (1 - e^{-(\alpha x + \beta x^{\gamma})})^{a}]^{b},$$

$$h(x) = \frac{ab(\alpha + \beta\gamma x^{\gamma-1}) e^{-(\alpha x + \beta x^{\gamma})} [1 - e^{-(\alpha x + \beta x^{\gamma})}]^{a-1}}{[1 - (1 - e^{-(\alpha x + \beta x^{\gamma})})^{a}]}$$

Corresponding quantile function is given by

$$Q(u) = F^{-1}(u) = -\frac{\ln[1 - [1 - (1-u)^{\frac{1}{b}}]^{\frac{1}{a}}]}{\alpha + \beta}$$

In this study, maximum likelihood estimation and inference are also discussed. Log-likelihood function is derived by

$$\log L = n\log a + n\log b - \alpha\sum_{i=1}^{n} x_i - \beta\sum_{i=1}^{n} x_i^{\gamma} + \sum_{i=1}^{n}\ln(\alpha + \beta\gamma x_i^{\gamma-1})$$
$$+ (a-1)\sum_{i=1}^{n}\ln(1 - e^{-(\alpha\sum x_i + \beta\sum x_i^{\gamma})})$$
$$+ b\sum_{i=1}^{n}\ln[1 - (1 - e^{-(\alpha\sum x_i + \beta\sum x_i^{\gamma})})^{a}]$$

The log likelihood function is maximized by solving nonlinear linear likelihood equations obtained by differentiating log likelihood function. Then, the first order partial derivatives with respect to three parameters are

$$\frac{\partial \log L}{\partial a} = \frac{n}{a} + \sum_{i=1}^{n}\ln(1 - e^{-(\alpha\sum x_i + \beta\sum x_i^{\gamma})})$$
$$+ b\sum_{i=1}^{n}\frac{-(1 - e^{-(\alpha\sum x_i + \beta\sum x_i^{\gamma})})^{a} - \ln(1 - e^{-(\alpha\sum x_i + \beta\sum x_i^{\gamma})})}{1 - (1 - e^{-(\alpha\sum x_i + \beta\sum x_i^{\gamma})})^{a}}$$

$$\frac{\partial \log L}{\partial b} = \frac{n}{b}$$

Then, the MLEs of the parameters are obtained by setting these above equations to zero and solve them simultaneously. For this aim, the R Project is used.

Application

Landslides are significant natural hazards in Turkey. 25% of country area is exposed to landslide hazard and 11% of total population is located in landslide areas. From 1900 to 2013 landslides affected 500 settlements and killed more than 200 people. Furthermore, more than 65000 dwelling units were relocated to safer places in this period.

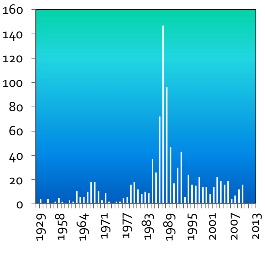

Figure 1. The number of landslides in Turkey

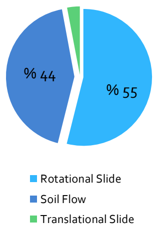

Figure 2. Type of landslides

The landslide data of Turkey is derived from the EM-DAT International Disaster Database maintained by the Center for Research on Epidemiology of Disasters (CRED) in cooperation with the United States Office for Foreign Disaster Assistance (OFDA). The database defines a destructive landslide as meeting at least one of the following criteria: 10 or more deaths, 2000 or more affected people, a government disaster declaration. In this study, the time between two consecutive landslides is shown by the random variable X. It is considered that X can follow KGEW, exp-Weibull, Weibull or gamma distribution. Then, parameter estimations of these distribution are obtained by R project. Maximum likelihood estimations and standard errors of the distributions are presented in Table 1.

Table 1. MLE and standard errors of the distributions

Distribution	Estimation				
KGEW (a, b, γ, α, β)	0.011 (0.009)	2.849 (0.459)	0.182 (0.025)	1.056 (0.002)	1.442 (1.378)
Exp.-Weibull (γ, α, β)	4.429 (1.025)	0.017 (0.025)	1.238 (0.901)	-	-
Weibull (γ, β)	2.789 (0.237)	0.024 (0.008)	-	-	-
Gamma (η, σ)	3.513 (0.867)	7.564 (1.196)	-	-	-

Cramer-Von-Mises (W) and Anderson Darling (A) statistics are used to decide the best distribution for the landslide data of Turkey. Then, the results are shown in Table 2.

Table 2. W and A Statistics

Distribution	W	A
KGEW (a, b, γ, α, β)	0.064	0.302
Exp.-Weibull (γ, α, β)	0.536	1.476
Weibull (γ, β)	0.259	1.235
Gamma (η, σ)	0.345	1.380

As seen in Table 2, the proposed KGEW distribution correspondence well to landslide data. Then, the survival function S(X) is presented in Figure 3.

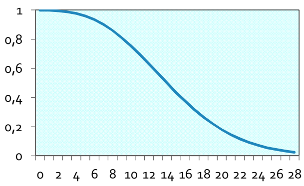

Figure 3. Survival Function for the Landslide Data

Conclusion

In this study, KGEW distribution is proposed and probability density, survival and hazard functions are provided. Then, parameter estimations are provided by ML method. The KGEW distribution is applied to landslide data of Turkey and correspond well this data.

References

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