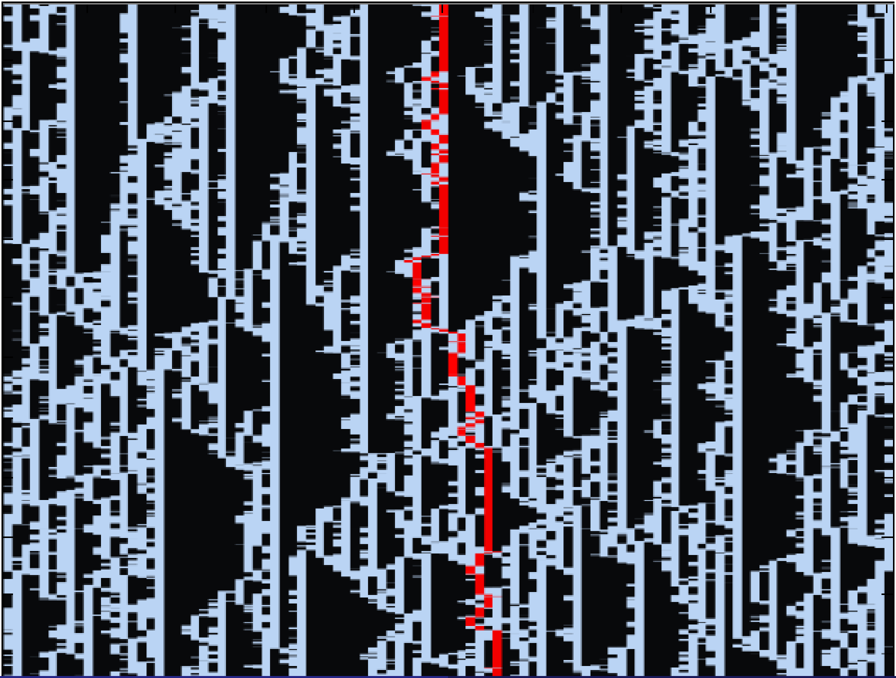
Relaxation and diffusion in the East model

Oriane Blondel
LPMA – Paris 7; ENS Paris

Young Women in Probability – Bonn
May 27th, 2014



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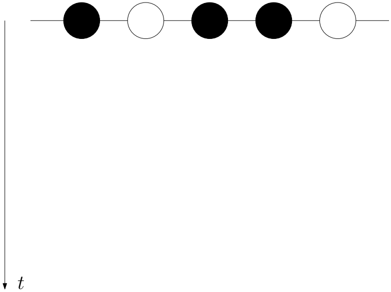

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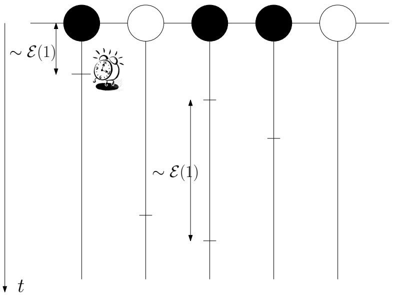

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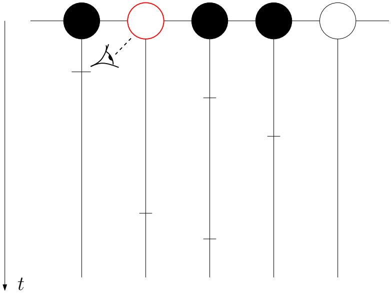

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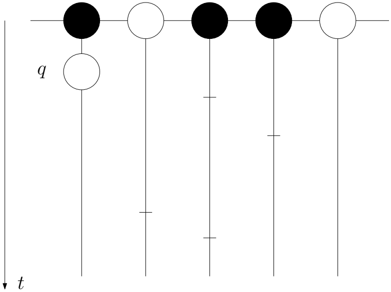

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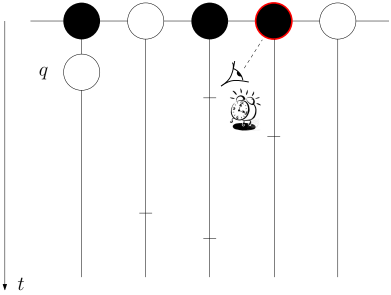

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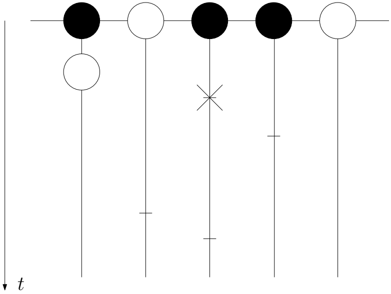

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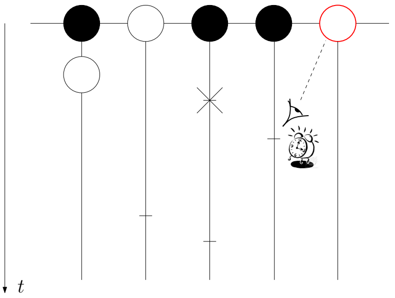

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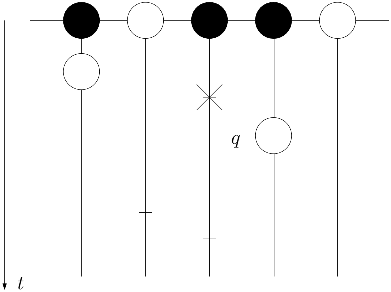

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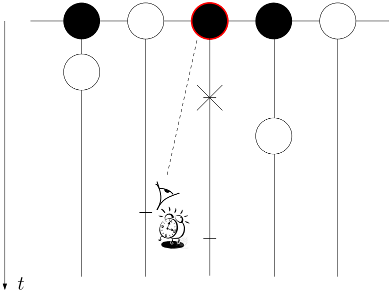

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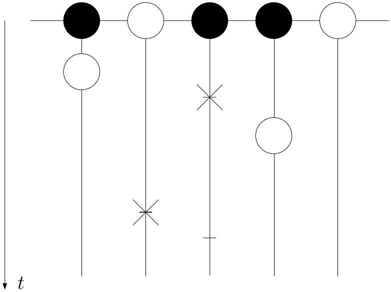

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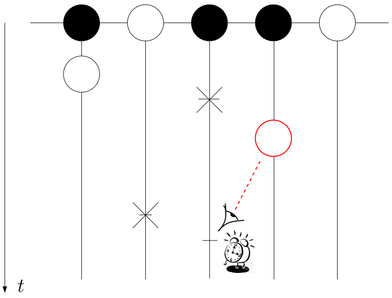

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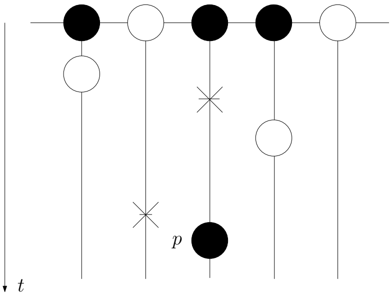

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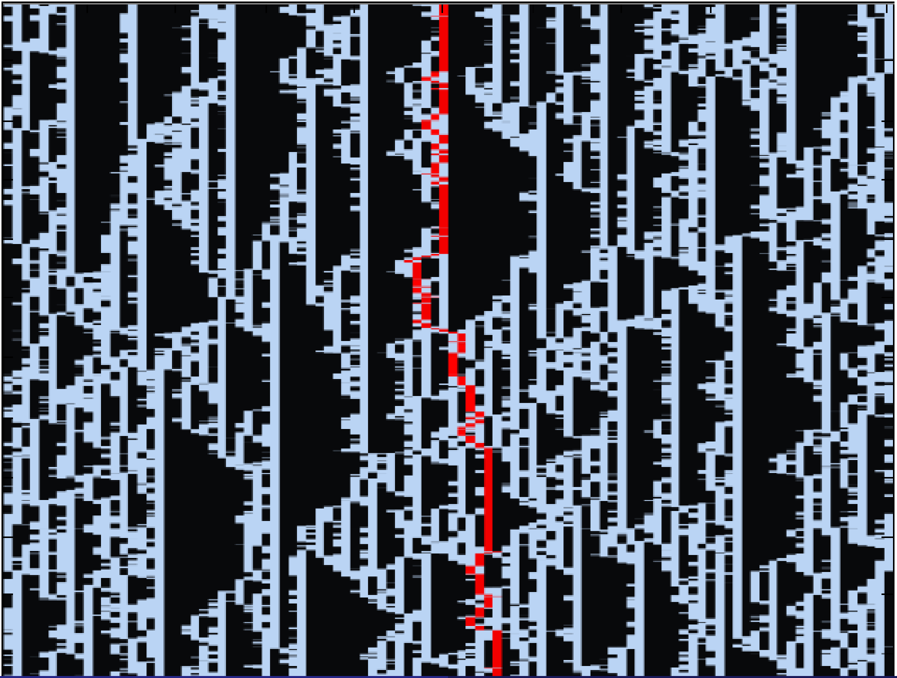

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- Exponential decay of correlations at equilibrium [Aldous-Diaconis '02]

$$Var_\mu(P_t f) \leq Var_\mu(f) e^{-2t/\tau} \quad \text{avec} \quad \tau < \infty,$$

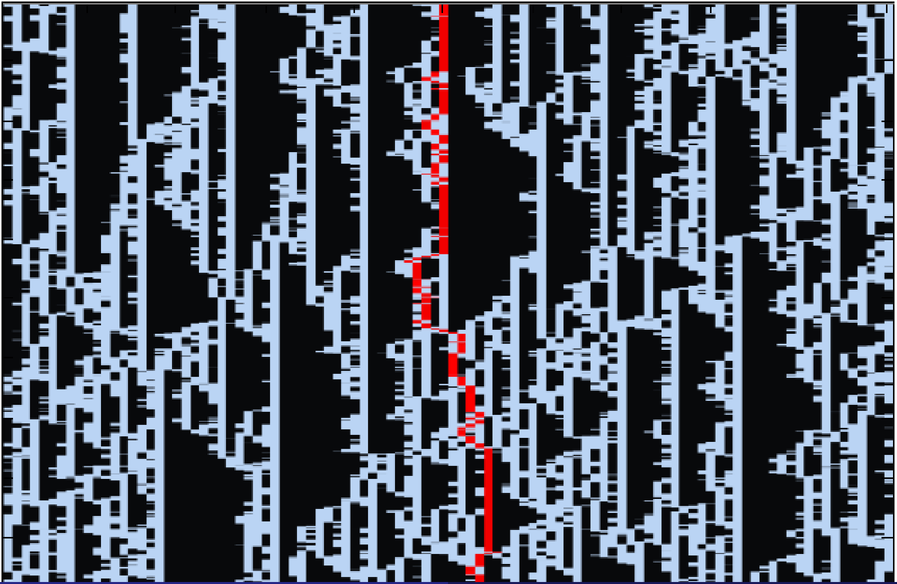
or

$$|\mu(f \cdot P_t g) - \mu(f)\mu(g)| \leq C_{f,g} e^{-t/\tau}.$$

"The correlation between ω and $\omega(t)$ decays as $e^{-t/\tau}$ when starting from equilibrium". τ is the *relaxation time* of the dynamics.

N.B.: $\tau = \tau(q)$.

Probing the bubble landscape



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- Environment: equilibrium East dynamics (initial configuration $\sim \mu$).
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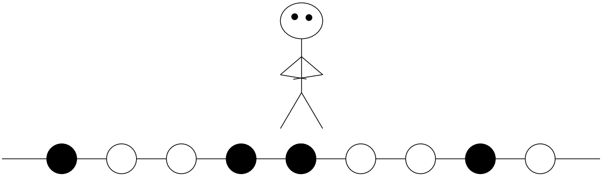

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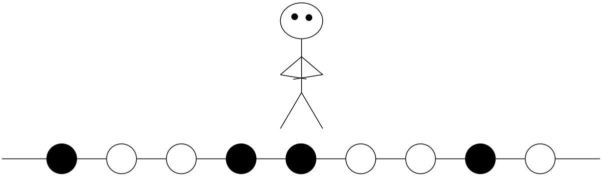

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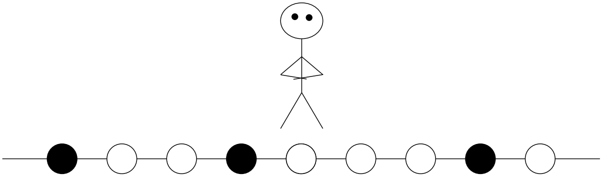

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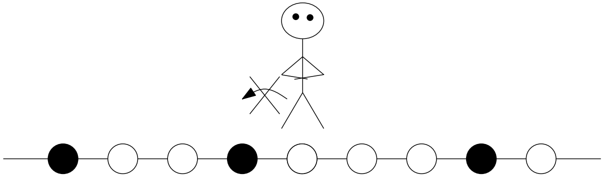

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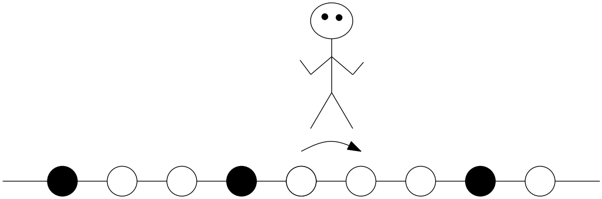

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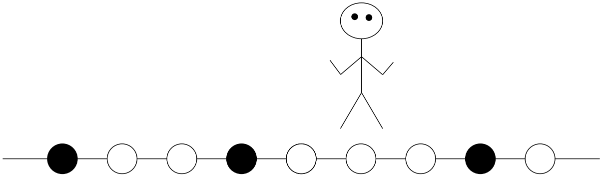

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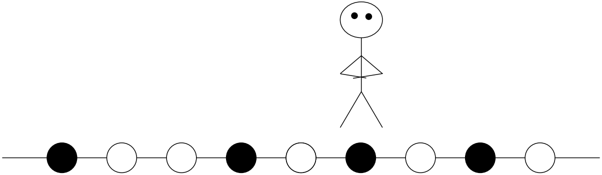

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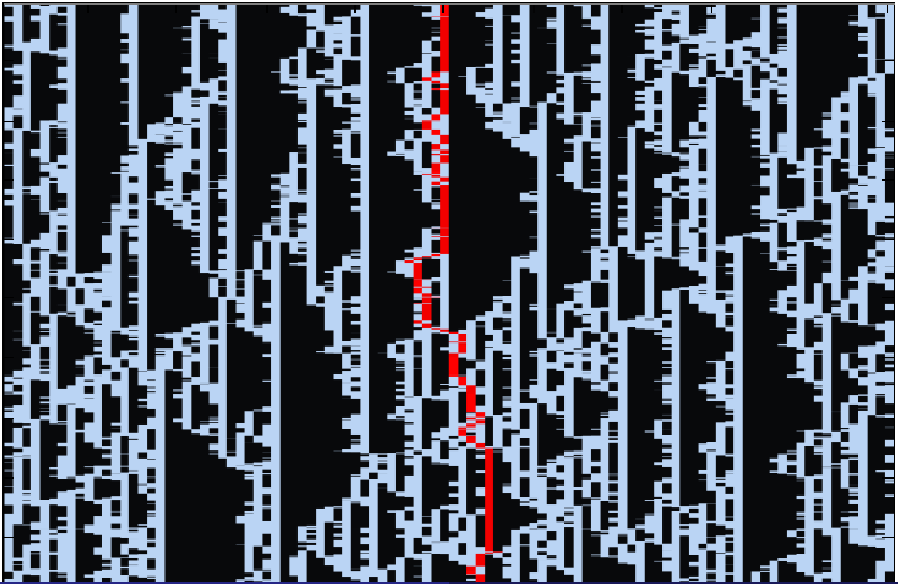


Diffusion coefficient

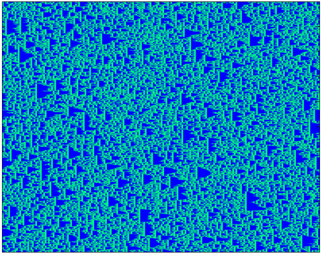
Under diffusive scaling, the tracer trajectory converges to a Brownian motion.
X_t: position of the tracer at time t.

$$\epsilon X_{\epsilon^{-2}t} \underset{\epsilon\to 0}{\longrightarrow} \sqrt{2D} B_t,$$

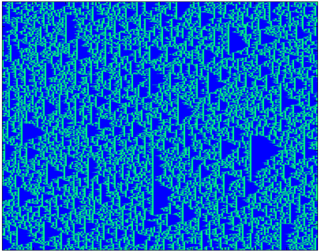
where $(B_t)_{t\geq 0}$ is a standard Brownian motion.

NB:

- Interaction with the environment encoded in the *diffusion coefficient* D.
- $D = D(q)$.



$q = 0.5$



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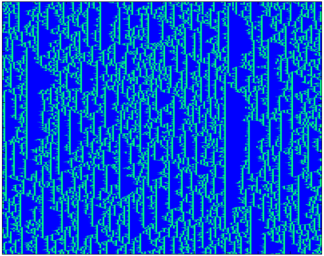
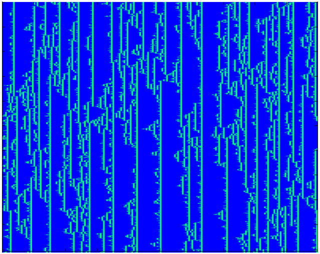


$q = 0.3$



$q = 0.2$

Simulations by Arturo L. Zamorategui.

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- Theorem [B. '13]
  There exist $C, \alpha < \infty$ not depending on q such that

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- [Jung-Kim-Garrahan-Chandler '13] New extended simulations, compatible with either $D \approx \tau^{-0.77}$ or $D \approx q^{-1.6}\tau^{-1}$.

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Combinatorial game [Chung-Diaconis-Graham '01].

- Board: $\mathbb{Z}^-$ with fixed zero at the origin.
- Game tokens: n zeros.
- Rule: zeros can be added to or removed from the board (except the fixed initial one), as long as the East constraint is respected (*i.e.* if there is a zero to the right.
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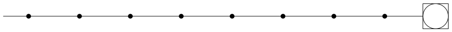

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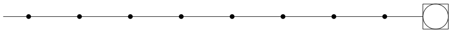

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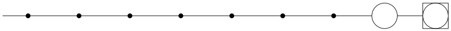

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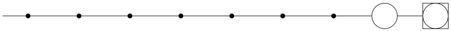

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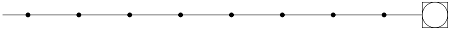

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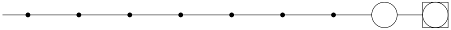

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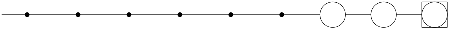
- Goal of the game: bring a zero as far left as possible.



$$n = 2$$

$$L_1 = 1 \quad ; \quad L_2 = ?$$

Heuristic for τ

Combinatorial game [Chung-Diaconis-Graham '01].

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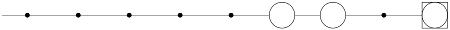

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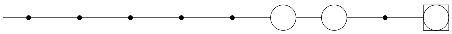

$n = 2$

$L_1 = 1 \quad ; \quad L_2 = 3$

Heuristic for τ

Combinatorial game [Chung-Diaconis-Graham '01].

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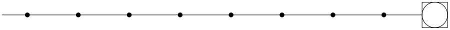

$$n = 3$$

$$L_1 = 1 \quad ; \quad L_2 = 3 \quad ; \quad L_3 = ?$$

Heuristic for τ

Combinatorial game [Chung-Diaconis-Graham '01].

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- Game tokens: n zeros.
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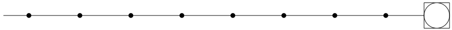

$L_1 = 1$; $L_2 = 3$; $L_3 = 7$ homework!

Heuristic for τ

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- Goal of the game: bring a zero as far left as possible.

Results:

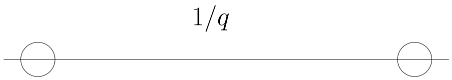
- $$L_n = 2^n - 1.$$

  In particular, we need $\approx \log_2 L$ zeros to cross a distance L.
- Number of configurations attainable with n zeros $\approx 2^{\binom{n}{2}} n! c^n$.

Heuristic for τ

$$\tau \approx (1/q)^{\log(1/q)/(2\log 2)}, \quad q \to 0.$$

- Under μ, typical configuration $\to$ isolated zeros at distance $1/q$.
- Consequence of the previous game: to cross a distance $1/q$, we need $\log_2(1/q)$ zeros.



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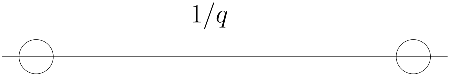

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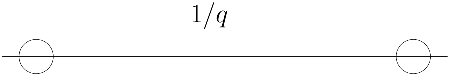
- The time it takes to cross $1/q$ is

$$\approx \left[q^{\log_2(1/q)} \qquad\qquad\qquad \right]^{-1}$$



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- The results of the game further say that there are $\approx 2^{\binom{n}{2}} \approx (1/q)^{\log(1/q)/(2\log 2)}$ such attainable configurations.
- The time it takes to cross $1/q$ is

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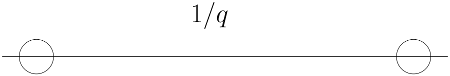

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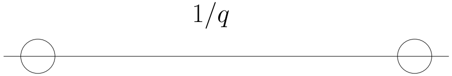
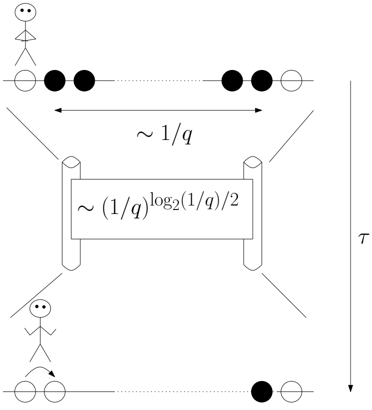
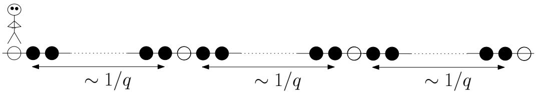
$$\approx \left[q^{\log_2(1/q)}(1/q)^{\log(1/q)/(2\log 2)}\right]^{-1} = (1/q)^{\log(1/q)/(2\log 2)}$$



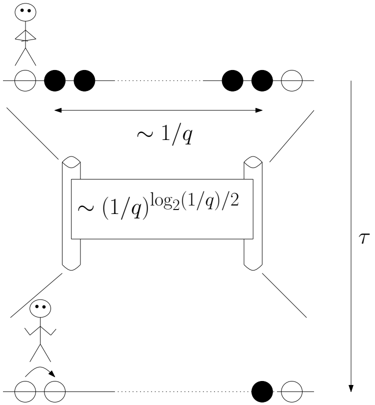
Consequences for D

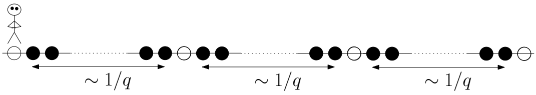




- $\mathbb{P}_\mu(X_\tau \geq 1/q) \ll 1$

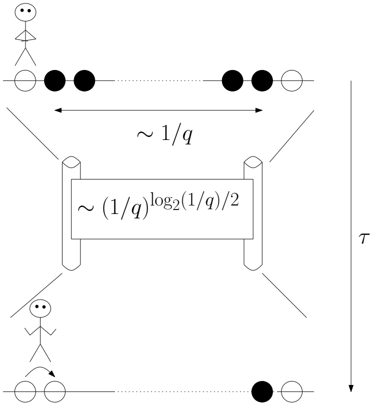
Consequences for D





- $\mathbb{P}_\mu(X_{q^\beta \tau} \geq 1/q) \leq Cq$
  Note: we need more precise estimates than the ones given by the game above; better bottleneck in [Chleboun-Faggionato-Martinelli '14].

Consequences for D



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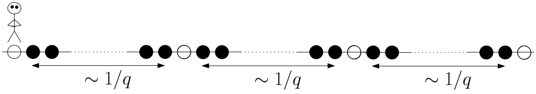
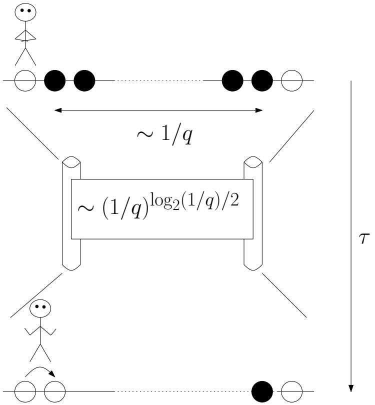
- $\mathbb{E}_\mu[X^2_{q^\beta \tau}] \leq Cq^{-C}$



Consequences for D



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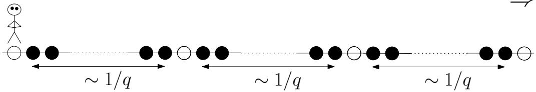
- $\mathbb{E}_\mu[X^2_{q^\beta \tau}] \leq Cq^{-C}$
- Mixing of the environment seen from the tracer on time scale τ

$$\Rightarrow \limsup_{t \to +\infty} \frac{1}{t} \mathbb{E}_\mu[X_t^2] \leq Cq^{-\alpha} \tau^{-1}$$



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Thank you for your attention!