Topics in Analysis and PDE V5B1

Coagulation-fragmentation equations

SS 2012, Do 12-14 h, Fr 10-12 h , SR 2.040

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In this course we will discuss several basic models that describe clustering phenomena as they occur, for example, in polymerization, the formation of liquid drops in oversaturated gases or the formation of galaxies. We will be interested in a continuum description via rate equations that leads to either systems of coupled nonlinear ODE or nonlinear integral equations.
In this course we will mainly consider two classical examples: first, the Becker-Döring system where it is assumed that clusters can only gain or shed one monomer at a time. Second, we will consider Smoluchowski's classical coagulation equation (fragmentation is not taken into account in this model) that describes the evolution of a density $f(x,t)$ of clusters of size x. The evolution is determined by the integral equation

$$\frac{\partial}{\partial t} f = \frac{1}{2}\int_0^x K(x-y,y)f(x-y,t)f(y,t)\,dy - f(x,t)\int_0^\infty K(x,y)f(y,t)\,dy\,.$$

Here, $K = K(x,y)$ is a coagulation kernel that describes the rate at which clusters of size x and y coagulate to a cluster of size $x+y$.
Topics that we will discuss are well-posedness of the corresponding initial value problems, existence and non-existence of equilibria, self-similar solutions, large-time behaviour and the phenomenon of gelation.

References

[1] J. Ball, J. Carr and O. Penrose, 1986, The Becker-Döring cluster equations: basic properties and asymptotic behaviour of solutions, *Comm. Math. Phys.* **104**, 657-692

[2] M. Escobedo, S. Mischler and B. Perthame, 2002, Gelation in coagulation and fragmentation models, *Comm. Math. Phys.* **231** , 157-188

[3] P. Laurençot and S. Mischler, 2004, On coalescence equations and related models, *Modeling and computational methods for kinetic equations*, Eds. P. Degond, L. Pareschi, G. Russo, Series Modeling and Simulation in Science, Engineering and Technology, Birkhäuser.

[4] G. Menon and R. L. Pego, 2004, Approach to self-similarity in Smoluchowski's coagulation equations, *Comm. Pure Appl. Math.* **57** 9, 1197-1232.

Prerequisites: Basic knowledge of Nonlinear Differential Equations and Functional Analysis, in particular weak convergence and weak compactness