

# Analytic aspects of statistical mechanics: Coulomb gases and Ginzburg Landau vortices Graduate Seminar (S4B1)

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**Topic** Consider  $N$  charged particles in  $\mathbb{R}^3$  that repell each other according to Coulomb's law while confined by a background potential  $V$  (the so-called three-dimensional Coulomb gas). They can be described by the energy functional

$$H_N(x_1, \dots, x_N) = \sum_{i \neq j} \frac{1}{|x_i - x_j|} + \sum_i V(x_i).$$

In statistical mechanics one then considers the Gibbs measure at temperature  $\beta^{-1}$  given by

$$d\mathbb{P}_{N,\beta}(x_1, \dots, x_N) = \frac{1}{Z_{N,\beta}} e^{-\beta H_N(x_1, \dots, x_N)} dx_1 \dots dx_N.$$

This poses several questions:

- Do minimizers of  $H_N$  exist? What do they look like on a macroscopic and microscopic level?
- Can we take the limit  $N \rightarrow \infty$  in some sense? Is there a limit functional  $H_\infty$ ?
- What does a typical sample from  $d\mathbb{P}_{N,\beta}(x_1, \dots, x_n)$  look like?

In this seminar we will try to answer some of these questions for the Coulomb gas. We will also see that similar models appear at several places, in particular we will discuss the Ginzburg-Landau model for superconductivity. Along the way we will encounter potential theory,  $\Gamma$ -convergence, and large deviation principles.

**Prerequisites** Knowledge of measure theory, functional analysis and elliptic partial differential equations (e.g. participation in the Bachelor course 'Functional analysis and partial differential equations')

**Preliminary meeting** Tuesday, January 29 at 14:15 in room 2.025.

**Time and Location** Mondays, 14-16, room 0.006

**Further information** Contact us by email: buchholz@iam.uni-bonn.de  
Also feel free to contact us if you would like to attend but you have a scheduling conflict.

## References

- [1] Sylvia Serfaty. *Coulomb gases and Ginzburg-Landau vortices*. Zurich Lectures in Advanced Mathematics. European Mathematical Society (EMS), Zürich, 2015.
- [2] Thomas Leblé and Sylvia Serfaty. *Large deviation principle for empirical fields of log and Riesz gases*. Invent. Math. 210 (2017), no. 3, 645-757
- [3] Sylvia Serfaty. *Systems of points with Coulomb interactions*. Plenary lecture at the ICM 2018, <https://www.youtube.com/watch?v=n1csbILFCpY>