

Nonlinear Partial Differential Equations I.

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Exercises: Marco Bonacini.

Outline: The goal of the course is to present some of the main techniques available to study the well-posedness and the qualitative behaviour of Nonlinear Partial Differential Equations. Some of the topics covered will include the Calculus of Variations, Regularity Theory for Elliptic Problems and well-posedness of Parabolic and Hyperbolic Equations.

The course will combine the study of general methods for proving existence, uniqueness and general properties of the solutions with the study of specific examples that will illustrate how to obtain insight about the general results as well as the limits of validity of the general methods.

Program:

(1) Elliptic Problems. Functional Analysis Techniques. Variational Methods. Weak Solutions. C^α -Regularity Theory for scalar equations (De Giorgi-Nash Theorem). Introduction to Regularity Theory for elliptic systems.

(2) Elliptic Problems. Critical points for variational problems (Mountain Pass Theorem and Applications). Non-variational methods. Fixed Point Theorems.

(3) Parabolic and Hyperbolic Problems. Galerkin Method. Semilinear Parabolic Equations.

Bibliography:

L. C. Evans, Partial Differential Equations. American Mathematical Society, 1991.

M. Giaquinta, Introduction to regularity theory for nonlinear elliptic systems, 1993.