

## Markov Processes

### Exercise sheet 12 from 07/17/2009

#### Exercise 1: Binomial tree model (10 points)

Consider a financial market with only one underlying asset, denoted by  $\{S_t^1 : t \in [0, T]\}$ , and an interest rate  $\{S_t^0 = e^{rt} : t \in [0, T]\}$  for bank investments. Let  $0 = t_0 < \dots < t_k = T$  with  $t_i := \frac{i}{k}T$  be a time discretization of the interval  $[0, T]$ . Suppose that at any of these time points it is possible to observe the course of the underlying and subsequently to reallocate the hedging portfolio (without investing any additional money). Furthermore suppose that there are only two developments possible within one period of time: Either  $S_{t_{i+1}}^1 = uS_{t_i}^1$  or  $S_{t_{i+1}}^1 = dS_{t_i}^1$  (with respectively positive probability), where  $0 < d < e^{r\frac{1}{k}T} < u$ .

- i) Calculate the hedging strategy for a general European option  $X = f(S_T^1)$  in the cases  $k = 1$  and  $k = 2$ . That is, determine the portfolio collocation backwards for any intermediate time point such that the final value  $X$  of the option is attained.
- ii) Indicate an algorithm to calculate the hedging strategy in the general case.

#### Exercise 2 : Ternary model (10 points)

Adopt the notations from exercise 1, but suppose now that the course of the underlying has a third possibility of development between any two time points. Take  $k = 1$  and show that there are European options that cannot be attained.

**HAVE A NICE SEMESTER BREAK !!!**