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Markov Processes

Exercise sheet 10 from 06/26/2009

Exercise 1: Chaos decomposition I (10 points)

Let $h \in L^2(T,\mu)$ and $F := \exp(W(h) - \frac{1}{2}\int_T h^2(t)\mu(dt))$, where $\{W_h\}_{h\in L^2(T,\mu)}$ is -as usual- a isonormal Gaussian process.

i) Calculate $D^m F$ for all $m \in \mathbb{N}$

ii) Determine the functions f_m appearing in the representation $F = \sum_{m=0}^{\infty} I_m(f_m)$.

Exercise 2 : Chaos decomposition II (10 points)

Let $\{W_t\}_{t\geq 0}$ be a standard one-dimensional Brownian motion. Determine the chaos decomposition (that means a representation of the form $F = \sum_{m=0}^{\infty} I_m(f_m)$ for suitable functions f_m) of the random variables $F_1 := \int_0^1 (t^3 W_t^3 + 2tW_t^2) dW_t$ and $F_2 := \int_0^1 te^{W_t} dW_t$.

Hint: For exercises 1 and 2, exercise 2 from sheet 9 might be useful.

Exercise 3: A product rule (10 points)

Let $F_1, F_2 \in \mathbb{D}^{1,2}$ such that F_1 and $\|DF_1\|_H$ are bounded. Show that $F_1F_2 \in \mathbb{D}^{1,2}$ and that the following product rule holds:

$$D(F_1F_2) = DF_1\ F_2 + F_2\ DF_1$$

Exercise 4: Another derivative... (10 points)

Let $h_1, \ldots, h_n \in H = L^2(T,\mu)$, $f \in \mathcal{C}_c^\infty(\mathbb{R}^n)$, $g \in \mathcal{C}_c^\infty(\mathbb{R})$ and $u \in \mathbb{L}^{1,2}$. Consider a cylindric function $F := f(W(h_1), \ldots, W(h_n))$, where $\{W_h\}_{h\in H}$ is -as usual- a isonormal Gaussian process.. Show that $Fg(\|u\|_H^2) \in \mathbb{D}^{1,2}$ and

$$D_t(Fg(\|u\|_H^2)) = D_tF\ g(\|u\|_H^2) + 2Fg'(\|u\|_H^2) \int_T u_s D_t u_s \mu(ds).$$